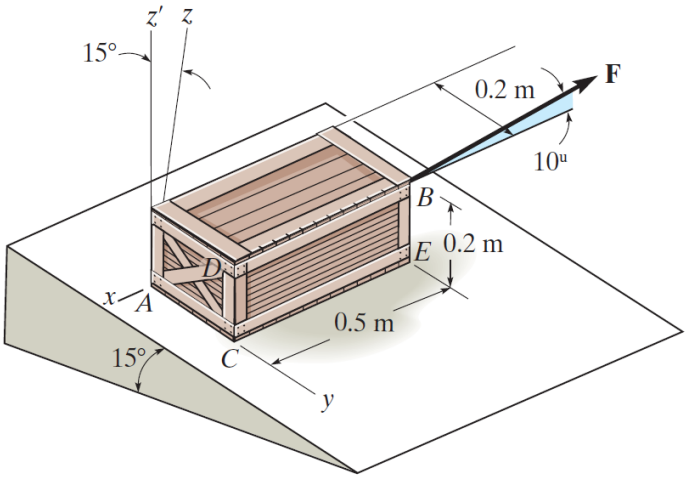
Problem 2-134

If the force $F = 100$ N lies in the plane $DBEC$, which is parallel to the x–z plane, and makes an angle of 10° with the extended line DB as shown, determine the angle that **F** makes with the diagonal AB of the crate.



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Solution

Write the position vectors to the points A and B.

$$\mathbf{r}_A = \langle 0, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle -0.5, 0.2, 0.2 \rangle \text{ m}$$

The unit vector going from A to B is

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} = \frac{\langle -0.5, 0.2, 0.2 \rangle}{\sqrt{(-0.5)^2 + (0.2)^2 + (0.2)^2}}.$$

Write the force in component form.

$$\mathbf{F} = 100 \langle -\cos 10°, 0, \sin 10° \rangle \text{ N}$$

Divide the force by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}}{|\mathbf{F}|} = \langle -\cos 10°, 0, \sin 10° \rangle$$

Take the dot product of the two unit vectors to get the angle between them.

$$\cos\theta = \frac{\mathbf{F}}{|\mathbf{F}|} \cdot \hat{\mathbf{u}}_{AB} = \langle -\cos 10°, 0, \sin 10° \rangle \cdot \frac{\langle -0.5, 0.2, 0.2 \rangle}{\sqrt{(-0.5)^2 + (0.2)^2 + (0.2)^2}} \approx 0.918$$

$$\theta \approx \cos^{-1}(0.918) \approx 23.4°$$